

Exercise 18

For the following exercises, use each set of functions to find $f(g(h(x)))$. Simplify your answers.

$$f(x) = x^4 + 6, \quad g(x) = x - 6, \quad \text{and} \quad h(x) = \sqrt{x}$$

Solution

Compute $f(g(h(x)))$ by plugging the formula for $h(x)$ where x is in the formula for $g(x)$ and then plugging this result into the formula for $f(x)$.

$$\begin{aligned} g(h(x)) &= (\sqrt{x}) - 6 \\ &= \sqrt{x} - 6 \end{aligned}$$

Therefore,

$$\begin{aligned} f(g(h(x))) &= (\sqrt{x} - 6)^4 + 6 \\ &= (\sqrt{x})^4 - 4(\sqrt{x})^3(6)^1 + 6(\sqrt{x})^2(6)^2 - 4(\sqrt{x})^1(6)^3 + 6^4 + 6 \\ &= x^2 - 24x^{3/2} + 216x - 864\sqrt{x} + 1296 + 6 \\ &= x^2 - 24x^{3/2} + 216x - 864\sqrt{x} + 1302. \end{aligned}$$